How Do Cultural Numerical Concepts Build Upon an Evolved Number Sense?

Helen De Cruz (hdecruz@vub.ac.be)
Centre for Logic and Philosophy of Science, Free University of Brussels, Pleinlaan 2
1050 Brussels, Belgium

Abstract

Several authors (e.g. Dehaene, 1997) have proposed that mathematical concepts build upon an evolved innate number sense, which we share with many other species. However, they have left unspecified how these mathematical concepts can come into existence and how they are transmitted. Models of content biased cultural transmission (e.g. Boyer, 2001) predict that cultural traits that bear a close fit to or a strong violation of intuitions provided by evolved cognitive mechanisms (modules) will be more widespread than others. Here, I adopt a similar approach to explain the emergence and spread of two mathematical concepts: positive integers, which have emerged independently in many cultures, and zero, which has evolved only once as a true numerical concept, but has successfully spread to other cultures. I will argue that these patterns of emergence and transmission lie in their fit with or their violation of intuitions provided by our evolved number sense.

Introduction

Many numerical concepts, like symbolic representations of positive integers, or ways to calculate surface areas, occur independently in widely varying cultures (Ascher, 1991). This makes mathematics more universal than science and even writing. Clearly, such complex concepts are the result of cultural evolution: a gradual and long accumulation of mathematical knowledge within different cultures. Nevertheless, recent evidence from animal, infant and neuro-imaging studies suggests that mathematical concepts are rooted in an evolved number sense that does not require cultural transmission. The relationship between cultural number concepts and this evolved numerical competence has not yet been fully explored.

In this paper, I shall examine the role of innately specified intuitions about number in the transmission of cultural numerical concepts. First, I shall discuss how evolved ways of conceptual thinking can influence cultural transmission. Next, I shall review the growing evidence for a conceptual number module in infant, animal, adult and neuro-imaging studies. Finally, I shall provide two case-studies, the positive integers and zero, to explain how this number module shapes and influences the emergence and transmission of cultural numerical concepts.

Conceptual modularity

Animals' brains enable them to behave adaptively, even in situations that they have not previously encountered. The main reason they are able to do this, is their ability to form conceptual thoughts. This is essential to any organism that engages in complex interactions with its environment. Without conceptual thought, an animal would perceive each object or event as unique, and generalizations would be impossible (Bovet & Vauclair, 1998).

Many models have been proposed to explain how the brain engages in conceptual thought. Evolutionary psychologists Cosmides and Tooby (1994) contend that the brain consists of many specialized systems - conceptual modules - that each have their own specific way to deal with a given subset of computational problems, in other words, they are domain-specific. Since problems encountered by humans and other animals are often mutually incompatible, natural selection has crafted a dedicated solution for each recurrent evolutionary problem. Such a conceptual module is not directly linked to perceptual input, but uses representations from perceptual modules (or other conceptual modules) as its input. By deriving information from other modules, conceptual modules enable the brain to achieve conceptual integration.

Conceptual modularity and culture

How can conceptual modularity explain the great diversity of human cognitive abilities? Most elements of human culture are too novel and too variable to be the specific output of any module. Even if we could provide adaptive explanations for some of these elements, this can never be true of all, simply because many of them differ widely across cultures. Sperber (1996) proposes to draw a distinction between proper and actual domains of modules. The proper domain is the reason a module exists, it is part of its evolutionary history, e.g. the proper domain of the face-recognition module is the human face. The actual domain is the set of stimuli to which the module responds, whether it belongs to its proper domain or not, e.g. the face-recognition module responds to *smileys* and masks as well.

Proper and actual domains of a module are unlikely to overlap entirely because cognition is a probabilistic activity. Natural selection can detect the formal properties of a stimulus to which the module responds, but this cannot rule out false negatives or false positives. Modules are mandatory: they are activated when the right type of stimulus is provided, even if this does not belong to the domain for which they have evolved. This slight mismatch between proper and actual domains takes place to a unique extent in human culture. Humans require much of their knowledge through cultural transmission. As a result, the actual domain of some modules can expand much beyond their proper domain (Sperber & Hirschfeld, 2004). Thus, cultural transmission results in a vast proliferation of domain-specific concepts, such as imaginary creatures or (as I shall argue) complex numerical concepts.

Content biases in cultural transmission

Cultural transmission is biased by innate predispositions. Since modules process information in a domain-specific manner, they influence how cultural information will be processed in the brain. This means that cultural traits are more successfully transmitted if they concur with basic intuitions of these conceptual modules, since modules are most likely to respond to cultural stimuli which have a strong concurrence with their proper domain. However, some cultural traits are widespread because they are counterintuitive: they are characterized by both violation of one module, and strong accordance to others (Boyer, 2001). Ghosts, for instance, have beliefs, desires and social interaction, in accordance with naïve psychology. On the other hand, they violate naïve physics because they can walk through walls or disappear at will.

Number as the proper domain of a conceptual module

Numbers - like other mathematical concepts - are abstract representations that seem far removed from elementary sense data. They appear to imply knowledge of language: arithmetic, counting and more complex computations involving number rely on linguistic tools. It seems therefore unlikely that number could constitute the proper domain of one or more dedicated modules. On the other hand, number is a basic property of the environment. From an evolutionary perspective, stable properties of the environment that yield potentially useful information can exert selective pressures on nervous systems in different animal species. Therefore, we can expect that modules that infer gravitational pull, colour, or numerosities occur in many species. Recognizing numerosities provides an animal with a mechanism to reduce complicated forms of input (objects or events in time and space) to simple numerical relationships.

Numerical competence in humans and animals

Current experimental evidence from different disciplines suggests that number is more than a cultural invention, and that human numerical competence has its roots in cognitive evolution. Three main lines of evidence support this claim. Numerical competence is present in human infants, prior to schooling or even language. Similar capacities have been found in non-human animals even in the absence of training. Furthermore, functional neuro-imaging studies strongly suggest that number processing rests on a distinct neural circuitry. Here, I will briefly review the evidence for the phylogenetic origins of humans' and other animals' numerical competence.

Infants share with other animals the capacity to reason about number. Newborns of just a few days old discriminate between sets of 2 and 3 items, but fail to see the difference between 4 and 6 (Antell & Keating, 1983). A much replicated experiment (Wynn, 1992) shows that five-month-olds can predict the outcome of an elementary addition or subtraction task for small numbers: they look longer at impossible (1 + 1 = 1) than at expected (1 + 1 = 2) outcomes. Infants can also keep track of small collections of items in working memory (e.g. 2 vs. 3) when these are presented serially, such as crackers being dropped in different opaque containers (Feigenson, Carey & Hauser, 2002).

This numerical ability is not restricted to small numerosities. In tasks involving large numbers of elements, infants compute discrete numbers as well. Six-month-olds dishabituate when the number of dots on a display changes from 8 to 16, even when other variables such as total surface area have been controlled (Xu & Spelke, 2000). They are capable of predicting outcomes of additions involving large numerosities, e.g. 5 + 5 = 10 and not 5 (McCrink & Wynn, 2004). This numerical ability apparently extends to different modalities and formats, including sounds (Lipton & Spelke, 2003) and actions (Wynn, 1998). This suggests that infants represent numerosities at a semantic level, i.e. as magnitudes, and not as a function of continuous variables such as density and duration.

Various vertebrate species have been demonstrated to show a sensitivity to number that is similar to that of human infants. Experiments using operant conditioning show that pigeons and rats are able to estimate large numerosities (key presses to obtain a food-reward), with increasing imprecision as the required number of presses gets larger (Brannon et al., 2001).

Experiments involving ecologically relevant stimuli provide evidence that animals spontaneously use numerical information from their environment to make adaptive decisions. Lionesses decide whether or not to attack a group of intruders (simulated as roars played on a tape) on the basis of the number of roaring lions they hear and the number of members of their own pride present (McComb, Packer & Pusey, 1994). When presented with two different numerosities of food-items, untrained free-ranging rhesus monkeys (Hauser, Carey & Hauser, 2000) and red-backed salamanders (Uller et al., 2003) go for the larger quantity.

Adults rely on numerical concepts similar to those of infants and animals when they are prevented from counting. If asked to press a key a specific number of times, while pronouncing 'the' (to prevent subvocal counting), the mean number of key presses increases in proportion to the target number (Cordes, Gelman & Gallistel, 2001). Adults' accuracy and speed in numerical performance gets worse as the absolute size of the numbers increase (size effect), and as the distance between them gets smaller (distance effect), e.g. a comparison between 7 and 8 typically takes a longer response time than between 3 and 8. This suggests they convert sets of discrete Arabic numerals to a continuous magnitude representation before the comparison process itself takes place (Moyer & Landauer, 1967). This mental number line has scalar variability: each number represents a tuning curve, which gets broader with increasing magnitude.

Thus, two large numbers will be harder to distinguish than two smaller numbers with the same absolute difference (Gallistel & Gelman, 2000). In sum, the mental number line makes approximate representations of number: infants, animals and adults who do not count can only make exact representations of the smallest quantities.

The neural architecture underlying numerical competence

The neural representation of number has been studied through various neuro-imaging techniques, including fMRI, PET and ERP. These have yielded a considerable body of converging evidence that numerical operations generate a specific pattern of brain-activation which recruits areas that are associated with linguistic tasks and visuo-spatial tasks (Dehaene et al., 1999). Models that invoke conceptual modularity at the neural level suggest that there are areas (conceptual modules) which are specialized in semantic-level processing of information of a specific domain. Such models have been proposed for other categories of semantic knowledge, e.g. plants and animals (Caramazza & Mahon, 2003). Several neuro-imaging studies indicate that the horizontal banks of the intraparietal sulci (HIPS) are the neural correlate of a conceptual module that deals only with numerical information. Its activation can be reliably dissociated from non-numerical tasks that require a similar level of attention, working memory, and spatial cognition (Simon et al., 2002). Its performance is not affected by the format or modality in which the task is presented (visual, auditory, as number words or as Arabic numerals) but depends on numerical complexity, such as absolute size and relative distance (Eger et al., 2003).

How can individual neurons recognize numerosities? Nieder, Freedman and Miller (2002) examined responses of individual number-sensitive neurons in rhesus monkeys to displays of 1 to 5 dots. These neurons only respond to numerical changes, and remain insensitive to changes in size or shape. Each neuron shows a peak activity to a specific quantity, and becomes progressively less active as magnitude increases. A neuron optimally activated by 2, is less responsive to 1 or 3, and even less so to more items. Thus, each number is represented by a set of neurons. Since neurons are only coarsely tuned, close numerosities will activate similar populations of neurons. Smaller numerosities yield narrower tuning curves, thus making them easier to distinguish, while larger numerosities yield broader tuning curves, which makes their discrimination fuzzier (Nieder & Miller, 2003).

The epidemiology of numerical concepts

Numerical concepts vary widely between cultures, yet some are more widespread than others. With a few exceptions (Pica et al., 2004) positive integers are universal, whereas reals occur rarely. Here, I shall argue that the emergence and transmission of numerical concepts (*in casu,* positive integers and zero) is influenced by their fit to or violation of conceptual modules, especially the number module.

The positive integers

Our mind represents reals and not integers. Some developmental psychologists (e.g. Wynn, 1998) believe that the positive integers {1,2,3,…} constitute the psychological foundation from which all other numerical concepts arise. However, since neurons are only approximately tuned to different numerosities, the magnitudes infants, animals and adults who do not count represent are more properly conceptualized as *reals* than as *integers*. Reals (e.g. π) are uncountable: they cannot be put in a one-to-one correspondence with natural numbers. We can conceptualize reals as being on a continuous scale, whereas the positive integers constitute a discrete set (Gallistel & Gelman, 2000). Therefore, any model that explains the success and salience of positive integers in various cultures has to explain why these are apparently intuitively appealing and culturally widespread, and reals are not.

The role of language in the development of integer concepts. To date, no language that completely lacks number words has been recorded. Nevertheless, there is considerable variation across cultures in their development: some cultures have elaborate positional number word systems, whereas others have a very limited number word vocabulary. Recent investigations in two indigenous South American hunter-gatherer societies show that some cultures do not even have true discrete numerical concepts. The Pirahã (Gordon, 2004) have only words for ‘one’, ‘two’ and ‘many’; sometimes using their word for ‘one’ for small collections of items. In controlled experiments they discriminate between very small sets such as {2, 3} while their performance for {3, 4} is at chance level. They seem to rely exclusively on an analog magnitude representation similar to infants and adults prevented from counting, instead of on positive integers. Like the Pirahã, the Mundurukú (Pica et al., 2004) are unfamiliar with counting. They also use number words (up to 5) in an approximate rather than precise fashion. For instance, when 5 dots are presented, the subjects respond ‘5’ in only 28 % of the trials, and ‘4’ or ‘few’ in 15 % of the trials. However, they are able to discriminate between sets of 20 and 80 dots.

These studies indicate that the mere presence of number words does not suffice to promote an exact representation of numerosities. More crucial perhaps is that both cultures lack a counting routine. Without the principle of one-to-one correspondence, a true understanding of the positive integers as discrete entities does not emerge, simply because the number-sensitive neurons’ representations are too fuzzy to enable discrete representations. Conversely, humans are able to make exact representations of magnitude without number words, e.g. a shepherd may keep track of the size of his flock by putting tallies into a one-to-one correspondence with his sheep as they move between shelter and pasture

(Ifrah, 1985). Thus, not language *per se*, but counting by sequential tagging is the key ingredient to a successful understanding of positive integers.

Counting through sequential tagging. The activity of counting involves a relationship between three sets: countable items, counting symbols and mental magnitudes (fig. 1a). The set of items to be counted varies from one count to another. To count, one establishes a one-to-one correspondence between each item and a conventionally defined list of symbols that have a fixed ordinality, e.g. the number words one, two, three (sequential tagging). The final item tagged determines the last tag from the counting sequence, which in turn denotes the cardinality of the set. This highest number word is mapped onto a corresponding mental magnitude. Through cultural learning, a synaptic connection gets established between these tags and the corresponding mental magnitude. Thus, the population of neurons in the HIPS that fires preferentially around 3 will respond to any symbolic representation for this magnitude. In this way, it becomes possible to establish a discrete, exact representation of magnitude by mapping an ordered set of symbols onto mental magnitudes. In contrast, the mental representation of approximate number words follows a different neural trajectory (fig. 1b). Here, cardinality is established directly, without sequential tagging. This approximate representation is subsequently converted into a linguistic expression, the approximate number word (e.g. few, a couple, about a hundred).

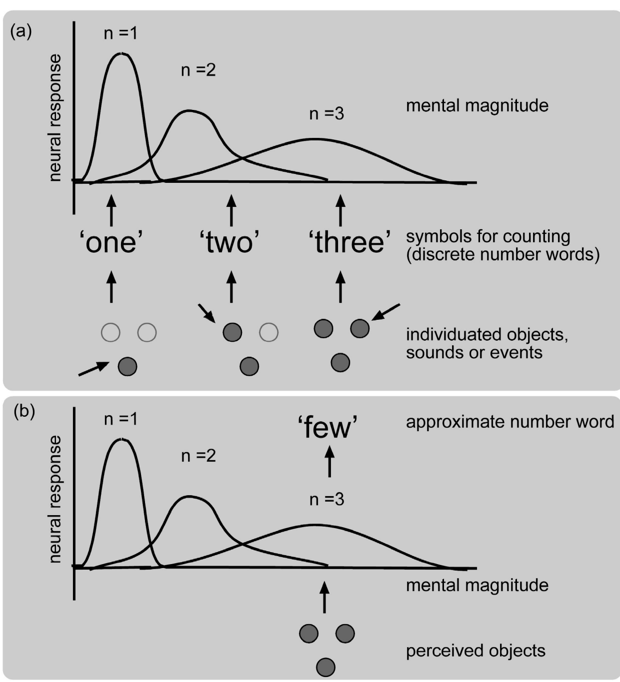

Figure 1: The linguistic representation of number in a (a) counting sequence and (b) approximately.

If counting is not an evolved ability, why has it emerged so frequently? The different steps involved in counting are subserved by neural circuits that lie anatomically close. The formation of new synaptic connections may be easier between two anatomically adjacent areas. To ensure that we count each item in a set exactly once, we shift our attention to each of them in a systematic, ordered way. This can be achieved by gesturing, pointing, or gaze direction. Indeed, under experimental conditions, children have more difficulties keeping track when they are prevented from gesturing or pointing (Alibali & DiRusso, 1999). The neural correlates of gesturing, pointing and visual attention lie very close to the HIPS (Simon et al., 2002). Culture may key in on this architectural property of the human brain by creating synaptic connections between them through learning how to count.

Similarly, establishing a one-to-one correspondence between an ordered list of numerical symbols and mental magnitudes often exploits the structure of the brain. Many cultures use body-parts (often fingers) as number symbols, which explains the wide occurrence of base-5, -10 and -20 positional systems (Ifrah, 1985). Neuropsychological evidence indicates that the identification of body-parts is a conceptual module, the 'body schema', whose impairment leads to a disability to identify one's own body parts. This module is an ideal candidate for a list of symbols with fixed ordinality (number symbols) because it also represents body parts in an ordered fashion: the comparison of body parts is prone to a distance effect similar to that in number comparison, e.g. to determine that the nose is lower than the eyes takes a longer response time than judging that the knees are lower than the eyes (Le Clec'H et al., 2000). The neural correlates of the body schema lie adjacent to the HIPS, in the left intraparietal lobule. A temporary disruption in this area (through rTMS) results in a marked increase in reaction-time when subjects complete a number comparison task, suggesting that finger counting continues to play an important part in adult numerical cognition (Sandrini, Rossini & Miniussi, 2004).

In brief, positive integers are not universal. They are not part of our number module, but are cultural constructions. Like many other cultural domains, they are rooted in more than one proper domain of conceptual modules (at least the modules for number, language, attention direction, body part identification). Indeed, most complex cultural concepts such as religion, art or mathematics are anchored in more than one conceptual module (Sperber & Hirschfeld, 2004).

Zero

The concept of zero as a numerical value has created opportunities in Western mathematics that could never have been realized without it. Zero extends to almost all domains of mathematics. It might therefore seem strange that zero is not widespread across cultures and their mathematical systems. From a content biased perspective on cultural transmission, however, this is not surprising. Neurons in

HIPS and prefrontal cortex fire preferentially at different numerosities, therefore different populations of neurons come to represent different numerosities. Zero obviously means an absence of neural activation in the number-sensitive areas of the brain. There is little point from an evolutionary perspective in having number-sensitive neurons that code for nothing. Since these areas are informationally encapsulated, they cannot reflect on their own lack of activation. Experiments (e.g. Wynn & Chiang, 1998) lend empirical support for this hypothesis. Eight-month-olds were confronted with either 'magical' or 'expected' events. In the 1 – 1 = 1 or 0 condition, they saw an object on a stage, a screen then occluded it, and a hand visibly removed it. Once the screen was removed, the infants either saw the expected result (no object) or a 'magical' appearance, in which the removed object was still there. Intriguingly, the subjects showed no surprise to this last result (1 – 1 = 1). This finding suggests the number module is not capable of representing zero, leaving infants unable to form the expectation that no objects will be seen.

How then did zero come into existence? Zero as by-product of a positional system that needed a symbolic representation for an empty place-holder emerged several times independently. However, the use of such an empty place-holder does not automatically lead to a true numerical concept for zero, as is aptly illustrated by Babylonian mathematics. At around 2000 BC, the positional system based on 60 emerged. Initially, an empty position was simply represented by a gap. Because this led to confusion, a separate symbol was invented to denote the empty space in the positional system. However, this symbol was never used in calculations. Since the Babylonian zero had no true numerical value, it clearly differs from that of Western mathematics (Ifrah, 1985).

Zero as a numerical value has its roots in classical Indian mathematics. It has been introduced to Western mathematics through Arabic mathematics. The oldest Indian mathematical concepts can be found in the Vedas, a collection of religious texts dating between 1500 and 500 BC, and the accompanying Vedangas, which contain *sutras* or rules that were of vital importance to the performance of ritual offerings. Large public offerings required altars constructed from complex geometric figures. If incorrectly constructed, the offerings would be of no value. Considering the effort, means and time invested in each public offering, its success was crucial. This increased selective pressure on mathematical concepts like geometry, arithmetic and (a precursor to) algebra. An additional selective pressure was provided by the fascination for large numerosities, salient in classical Indian poetry. It originated from stylistic considerations: since large numerosities impressed readers, poets felt compelled to use ever increasing magnitudes to emphasize the age, size or distance of any event, building or other thing they described. This led to the introduction of words that could express powers of ten. Combined with words that denote single digits up to nine, this enabled the elegant and parsimonious formulation of very large numbers. Parsimony was important for Vedic texts which were (and still are) learnt by heart. This eventually resulted in the invention of a positional system, in which a symbol for the empty place-holder became essential. However, it was only in Jain mathematics that zero became a fully-fledged mathematical concept (Joseph, 1990). As a reaction to Brahmin orthodoxy, Jains no longer practiced complex offerings. Severed from its religious origins, mathematics became a discipline studied for its own sake. The Jains had a fascination for mathematical concepts such as infinity, positing several types of infinite sets centuries before Cantor. Words for nothingness such as *shunya* meant more to them than absence or void, but implied receptiveness. Because of their cosmological ideas of time and space, emptiness could be conceptualized as a thing in its own right, instead of merely being an empty placeholder. This led Indian mathematicians like Brahmagupta (ca. 600 AD) to wonder how numerical operations with zero could be performed, e.g. whether or not one could divide by zero. This mature mathematical concept of zero subsequently spread to China, and the Islamic world, whence it diffused to the West.

The emergence of zero as numerical concept in Jain mathematics was possible because it could free-ride on certain cosmological and philosophical concepts. However, this does not explain its subsequent spread to other cultures. A possible explanation for this success is its violation of intuitive expectations of the number module. A numerical concept for nothing violates expectations provided by our evolved number sense. Cross-cultural experiments (Boyer & Ramble, 2001) show that people are prone to remember facts or narratives with an element that violates ontological expectations. Such counter-intuitive ideas are easier to remember than intuitive ideas (e.g. a creature that can be in more than one place at the same time versus a creature that has to eat to sustain itself) (Barrett & Nyhoff, 2001). Thus, zero as a numerical concept was long in the making because it is counter-intuitive. However, once originated, it becomes appealing because it violates ontological expectations.

Conclusion

In this paper, I have argued that the emergence and subsequent spread of cultural number concepts is influenced by the evolved structure of our brain. The multiple and frequent cultural invention of positive integers can be explained as a result of their close fit with intuitions provided by conceptual modules such as the number module. The anatomical proximity of this module to other modules involved in the counting by sequential tagging procedure further adds to its salience. The unique invention of zero as a numerical concept and its successful spread can also be explained as a result of content biased cultural transmission. Its counter-intuitiveness facilitates its cultural transmission; once it has emerged, it can easily spread to neighbouring cultures with positional systems.

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